Determine if each series below is convergent or divergent. **Do NOT find the sum.** **SCORE: _____ / 16 PTS**

[a] $\sum_{n=1}^{\infty} \frac{\cos n}{e^n - 1}$

SUBTOTAL $= 10$

(1) $0 \le \frac{|\cos n|}{e^n - 1} \le \frac{1}{e^n - 1}$ (1)

BY L'HÔPITAL'S

(1) $\lim_{n\to\infty} \frac{\frac{1}{e^n-1}}{\frac{1}{e^n}} = \lim_{n\to\infty} \frac{e^n}{e^n-1} = \lim_{n\to\infty} \frac{e^n}{e^n}$

OR $= \lim_{n\to\infty} \frac{1}{1-e^{-n}}$

$\Bigg\} = 1 \neq 0$ (1) (1)

(1) FOR EITHER METHOD

(1) $\sum_{n=0}^{\infty} \frac{1}{e^n}$ CONVERGES $(|r| = \frac{1}{e} < 1)$

SO $\sum_{n=0}^{\infty} \frac{1}{e^n - 1}$ CONVERGES BY LIMIT COMPARISON TEST (1)

SO $\sum_{n=0}^{\infty} \frac{|\cos n|}{e^n - 1}$ CONVERGES BY COMPARISON TEST (1)

SO $\sum_{n=0}^{\infty} \frac{\cos n}{e^n - 1}$ CONVERGES BY ABSOLUTE CONVERGENCE TEST (1)

[b] $\sum_{n=1}^{\infty} \frac{100^n n^{100}}{n!}$

$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{100^{n+1}(n+1)^{100}}{(n+1)!} \cdot \frac{n!}{100^n n^{100}} \right|$ (1)

SUBTOTAL $= 6$

$= \lim_{n\to\infty} \left| \frac{100}{n+1} \frac{(n+1)^{100}}{n^{100}} \right|$ (2)

(1) $= 0 < 1$ (1)

SO $\sum_{n=1}^{\infty} \frac{100^n n^{100}}{n!}$ CONVERGES (1)

Determine if $\sum_{n=2}^{\infty} \frac{(-1)^n \ln n}{\sqrt{n}}$ is absolutely convergent, conditionally convergent, or divergent. **SCORE: _____ / 12 PTS**

$$\sum_{n=2}^{\infty}\left|\frac{(-1)^n \ln n}{\sqrt{n}}\right| = \sum_{n=2}^{\infty} \frac{\ln n}{\sqrt{n}}$$

(2) $\underbrace{0 < \frac{1}{\sqrt{n}} < \frac{\ln n}{\sqrt{n}}}$ FOR $n > e$ (OR $n \geq 3$)

(1) $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}}$ DIVERGES ($p = \frac{1}{2} < 1$)

SO $\sum_{n=2}^{\infty} \frac{\ln n}{\sqrt{n}}$ DIVERGES BY COMPARISON TEST (1)

(1) $\frac{\ln n}{\sqrt{n}} > 0$ FOR $n > 1$ (OR $n \geq 2$)

$$\lim_{n\to\infty} \frac{\ln n}{\sqrt{n}} = \lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{1}{2\sqrt{n}}} = \lim_{n\to\infty} \frac{2}{\sqrt{n}} = 0 \quad (2)$$

$$\left(\frac{\ln x}{\sqrt{x}}\right)' = \frac{\frac{1}{x}\sqrt{x} - \frac{1}{2\sqrt{x}}\ln x}{x} = \boxed{\frac{\frac{1}{\sqrt{x}}\left(1 - \frac{1}{2}\ln x\right)}{x}} < 0 \text{ FOR } x > e^2 \quad (2)$$

SO $\left\{\frac{\ln n}{\sqrt{n}}\right\}$ IS DECREASING (1)

SO $\sum_{n=2}^{\infty} \frac{(-1)^n \ln n}{\sqrt{n}}$ CONVERGES (CONDITIONALLY) (1)
(1) BY ALTERNATING SERIES TEST

The terms of a series are defined recursively by the equations $a_1 = 5, \quad a_{n+1} = \frac{1-3n}{2n+1} a_n$. **SCORE: _____ / 4 PTS**

Determine whether $\sum_{n=1}^{\infty} a_n$ converges or diverges.

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{1-3n}{2n+1}\right| = \lim_{n\to\infty}\left|\frac{\frac{1}{n} - 3}{2 + \frac{1}{n}}\right| = \left|-\frac{3}{2}\right| = \frac{3}{2} > 1$$

(1) (1) (1)

SERIES DIVERGES (1)